

Prominence and flare fine structure from cross-field thermal conduction*

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Abstract

Thermal conduction across a magnetic field is strongly suppressed compared with conduction along the field. However, if a flare is heated by a highly filamented beam directed along the field, then the array of heated cells in a cross-section of the flare will result in both small spatial scales (with consequently large temperature gradients) and a large surface area for the heated volume, providing a geometrical enhancement of the total cross-field energy flux. To investigate the importance of this filamentary geometry, we present a simple model of a single heated filament surrounded by an optically thin radiating shell, obtain an analytical expression for the stable equilibrium temperature profile within the shell, and use this to impose limits on the size of filament for which this model is appropriate.

We find that this mechanism by itself is capable of transporting a power of the same order as a large flare, with a moderate range of filament sizes. The length scales are substantially smaller than can be resolved at present, although they should be regarded as underestimates.

Key words: Plasmas – Sun: flares – Sun: filaments

1 Introduction

There is an observational consensus (Engvold, 1976; Chiuderi Drago et al., 1992) that solar prominences have a good deal of fine structure within them, on scales ranging from 100 km to 1500 km; similarly, each improvement in resolution finds more structure within flares. Other studies, such as Fletcher and Brown (1995), provide indirect support for the existence of fine filamentary structure in flares, by invoking the presence of such structure to explain polarisation observations. At the same time, there

seems to be a theoretical consensus both that prominences condense due to a thermal instability (Field, 1965) in the plasma around them, and that cross-field thermal conduction plays a significant role in determining the scale of their fine structure (Van der Linden, 1993). However, these studies (as reviewed by (Heyvaerts, 1974)) concentrate on the dynamics of the instability which forms the threads, rather than on the final equilibrium of the resulting structure. In this paper, we instead present a simple, quasi-static, analytical model of a filamentary structure in a prominence or flare, which exhibits the small spatial scales necessary to produce a temperature gradient large enough to make cross-field thermal conduction significant, and which potentially gives cross-field conduction a dominant role in governing both the number and size of the threads in a prominence or flare, and the energy budget of the beam or current supplying the thread.

As discussed in a preliminary version of this model (Brown and Gray, 1994), we have available a large number of free parameters, so the aim has to be not to make a single prediction, but rather to see which combinations of parameters allow a consistent model.

The paper is organised as follows: in section 2 we describe the model we have used; in section 3 we describe the analytical and numerical solutions to this model that we have obtained; in section 4 we discuss the stability of these solutions; and in section 5 we briefly discuss the minor changes to our results which are necessary in the case of a low- β plasma.

2 The model

We suppose the existence of a heating mechanism which is spatially fragmented with filaments aligned along the ambient magnetic field, and so divides the heated plasma into a number of identical cylindrical cells, radius R , each of which is heated only within some coaxial cylindrical core (we have also considered the case of filamentation into

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plane sheets, with similar results to those presented here). This heated core is surrounded by a shell which is heated solely by conduction from the core, and cooled solely by radiation. We find the temperature structure of this shell which allows an equilibrium to form by radiating away all the energy that is transported into it. We can ignore the effects of longitudinal conduction, and assume that the filamentation allows the transverse conduction to dominate – this is retrospectively justified in Sect. 3.4. Thus, the equilibrium structure of a particular ‘slice’ (of length l) through the cell will be governed only by the power input at a particular point along the cylinder. When we wish to find numerical values, below, we will take $l = 10^7$ m as the cylinder length (this is the approximate stopping distance for a typical electron beam).

We have been deliberately vague about the heating mechanism – we will find that the only relevant parameter is the total power per unit length conducted from within the core, so that this heating mechanism could equally well be a non-thermal beam or a dissipating current. The cost of this generality is that we can say nothing about the structure of the core, nor fix what fraction of the total power available is conducted out, rather than being carried longitudinally within the core. We expect, however, that the heat-loss from the core will be substantial.

We parameterise distances from the centre of the cell in terms of the dimensionless parameter $s = r/R$, where $s = 1$ at the edge of the cell. We will take the shell to consist of material below some temperature T_ρ which we will take, below, to be around the peak in the Cox-Tucker radiative loss function. This shell starts a distance ρR from the centre of the cell, so that ρ is defined by

$$T(s = \rho) = T_\rho. \quad (1)$$

Conduction need not be the only method by which the core is cooled, but we will denote by P_ρ the power actually conducted into the shell per unit length along the field. Finally, we take each cell to be independent, so that there is no energy flux between cells; thus the temperature gradient must vanish at $s = 1$.

We emphasise that all the power input is confined within the radius ρ , and that we will not concern ourselves further with the details of this input nor the internal structure of the core. For this to be valid we must, for example, take the plasma in the shell to be optically thin so that it will not experience significant secondary heating by any radiative emission from the core.

In the shell, there will be two processes competing: there will be a conductive heat flux entering from the

core, and radiative emission from the heated plasma. For $\rho \leq s \leq 1$, the conductive heat flux per unit axial length is

$$P(s) = Q(s)2\pi sR, \quad (2)$$

where the heat flux density $Q(s)$ is (neglecting $\kappa_{\parallel}dT/dr$ along the beam)

$$Q(s) = -\kappa_{\perp}(s)\frac{d}{dr}T(s), \quad (3)$$

and the cross-field thermal conduction coefficient is (Rosenbluth and Kaufman, 1958)

$$\kappa_{\perp} = \kappa_1 \frac{n^2}{B^2 T^{1/2}} \quad (4)$$

with n in m^{-3} , B in Tesla, and

$$\kappa_1 \equiv 9.76 \times 10^{-41} \text{ W m}^5 \text{ T}^2 \text{ K}^{-1/2}$$

(note that the effective coefficient is likely to be rather larger than this – see the discussion in section 3.2 below). High in the solar atmosphere, magnetic pressure dominates, and we take the magnetic field to be approximately the same across the filament, so that the pressure-balance condition reduces to

$$n(s)T(s) = a, \quad (\text{constant}) \quad (5)$$

(but see Sect. 5) and in terms of $\kappa_0 \equiv \kappa_1/B^2$ we can write

$$\kappa_{\perp} = \kappa_0(B) \frac{n^2}{T^{1/2}} = \kappa_0(B) a^2 T^{-5/2}. \quad (6)$$

Given that the emitted power from a volume dV is $n^2 f(T) dV$, for a given annulus at radius s , we will have

$$P(s) - P(s + ds) - n(s)^2 f(T(s)) \cdot 2\pi s R \cdot R ds = 0,$$

which, on substitution of Eqn. (2), reduces to

$$\frac{d}{ds}[sQ(s)] + n^2 f(T) s R = 0. \quad (7)$$

The vanishing of the radial flux at the outer boundary of the slice provides one boundary condition $Q(1) = 0$, and another will be provided indirectly by the specified total power input to the shell at radius ρ .

Before going on to determine the temperature profile explicitly, it is worthwhile to point out that the forms of the transverse conduction coefficient (Eqn. (6)) and radiative loss function $f(T)$ conspire to produce a temperature profile which is radically different from the profile produced

by longitudinal conduction (as in the usual treatment of flares and the transition region). Because κ_{\perp} depends on a substantial negative power of the temperature, the thermal conductivity *improves* for lower temperature, in the sense that a shallower gradient is needed to carry a given heat flux. Between $T = 10^4$ K and 10^5 K the radiative loss function $f(T) \sim T^2$ and this decrease in the efficiency of radiative loss at lower temperature, along with the improvement in κ_{\perp} there, means that the temperature gradient is very shallow at the cold outer edge of the shell, and steepens particularly sharply at the heated inner boundary, quite unlike the profile in a normal gas or solid.

3 Analytical temperature profiles

The function $f(T)$ has an intricate dependence on temperature, but for our purposes we may take it to be a series of power laws (Rosner et al., 1978)

$$f(T) = \chi_i T^{\beta_i}. \quad (8)$$

In terms of the parameter

$$\tau(s) \equiv \frac{2}{3} T^{-3/2}(s), \quad (9)$$

the heat flux density is

$$Q(s) = \kappa_0 a^2 \tau'(s)/R. \quad (10)$$

With this form for $Q(s)$, and the above form for $f(T)$, we can rewrite Eqn. (7) as

$$\frac{d}{ds}(s\tau') + cs\tau^{-b} = 0, \quad (11)$$

in terms of a constant $c \equiv (3/2)^{-b} \chi(RB)^2/\kappa_1$, the parameter $b \equiv 2(\beta - 2)/3$, and boundary condition $\tau'(1) = 0$.

This does not seem to be analytically solvable in general, but for the temperature range below 10^5 K, we can set $\chi = 10^{-44} \text{ W m}^3 \text{ K}^{-2}$ and $\beta = 2$ in Eqn. (8). For $\beta = 2$, we have $b = 0$, and Eqn. (11) is easily solved to give

$$s\tau' = \frac{c}{2}(1 - s^2) \quad (12)$$

$$\tau = \tau(\rho) + \frac{c}{2} \left(\ln \frac{s}{\rho} + \frac{1}{2}(\rho^2 - s^2) \right), \quad (13)$$

with $\tau(\rho) = \tau(T_{\rho})$.

3.1 Consequences of these solutions

We may make a number of observations about these solutions. The first is that, if we define $\delta = 1 - \rho$, we can rewrite Eqn. (13) as

$$(R\delta B)^2(1 + O(\delta)) = \frac{2\kappa_1}{\chi}(\tau(1) - \tau(\rho)), \quad (14)$$

and find that the radial thickness of the shell, $R\delta$, is, to $O(\delta)$, dependent only on the magnetic field and the temperature at the edge of the cell $\tau(1)$ (where $\tau(1) \gg \tau(\rho)$, or $T(1) \ll T_{\rho}$). For $T(1) = 10^4$ K, Eqn. (14) produces $R\delta B \approx 0.12$, giving a shell thickness of only 12 m for $B = 10^{-2}$ T.

The total power removed from a single core in length l is

$$P_{\rho} = 2\pi\rho RlQ(\rho). \quad (15)$$

If we have a large number of such filaments, each of radius R , making up a total cross-sectional area of A , and we write P_A for the total power input across this area, then we can rewrite Eqn. (15) with the help of Eqn. (10) to find

$$\frac{P_A}{lA} = \chi a^2(1 - \rho^2). \quad (16)$$

This indicates two things. Firstly, since ρ has a minimum of zero, there is a maximum to the power that can be removed from the core by conduction into the shell (namely $P_A = 10^{22}$ W for $l = 10^7$ m, $A = 10^{13}$ m² and $a = 10^{23}$ K m⁻³) and this maximum is of similar order to the power of a large flare. It is difficult to know if this is a coincidence or not: noting that it corresponds to the central core shrinking to a line, it seems likely that it merely reflects the observation that there is an upper limit to the power a given volume of plasma can radiate, and that Eqn. (16) is independent of n and T only because we have approximated $f(T) \propto T^2$ to obtain the solution which produced it, making the losses $n^2 f(T)$ a constant. It is nonetheless true that $n^2 f(T)$ is approximately constant below 10^5 K, so that Eqn. (16) retains a qualitative general validity.

Secondly, Eqn. (16) is independent of the cell radius R . This is initially surprising, as one would expect that as the number of cells is increased and R falls, the ratio of the conducting surface area to the radiating volume would increase with $1/R$ and a greater power could be removed for a given value of ρ . However, since τ' scales like R^2 , the flux density $Q(\rho)$ itself scales like R , and so the total flux removed from the core scales with the same power of R as the radiating volume.

What is happening is that the temperature gradient at the surface of the core becomes steeper as the volume of shell it has to supply increases, and this steepening allows us to put limits on the permissible values of R . If the temperature gradient becomes too great, then normal conduction will be replaced by a more efficient anomalous mechanism which will effectively reimpose the maximum gradient allowed by normal conduction – the plasma will have this gradient for some distance from the surface of the core, until the solution Eqn. (13) takes over, with a suitably modified inner boundary condition, but the same outer one. The discussion below formally considers limits on the model's validity, but since the maximum-gradient solution is unlikely to be stable, it describes physical limits as well.

To establish the limits on the radius in the absence of this effect, we must insist first of all that the shell width is larger than the ion-gyroradius,

$$R\delta \gtrsim R_p \quad (17)$$

where

$$R_p \approx 10^{-6} \frac{T_\rho^{1/2}}{B}, \quad (18)$$

(in the numerical expressions here and below, B is in Tesla, R in metres, and n in m^{-3}). Comparison with Eqn. (14) shows that Eqn. (17) is comfortably satisfied, independently of B , for $T \lesssim 10^6$ K. Secondly we can define a length-scale for the gradient, l_T , via $dT/dr \sim T_\rho/l_T$ so that, using the solution in Eqn. (12) and the definition of τ in Eqn. (9), we have

$$l_T \sim \frac{2 \times 10^4 \rho}{T_\rho^{3/2} R B^2}, \quad (19)$$

and the condition $l_T \gtrsim R_p$ results in

$$R \lesssim 2 \times 10^{10} \frac{\rho}{B T_\rho^2}. \quad (20)$$

A third length scale is provided by the ion mean free path

$$R_f \approx 2 \times 10^8 \left[\frac{T_\rho^2}{n} = \frac{T_\rho^3}{a} \right], \quad (21)$$

independently of the magnetic field. Imposing $l_T \gtrsim R_f$ results in

$$R \lesssim 10^{19} \frac{\rho}{B^2 T_\rho^{9/2}}. \quad (22)$$

Together, these various constraints require that the radius R lies between the solid lines (1) to (4) in Fig. 1.

Figure 1 is in file figr1.ps

Figure 1: The restrictions on the cell radius R , obtained by comparing the temperature gradient with the ion gyro-radius and ion mean free path. See text for explanation. For comparison, line 5 corresponds to the scale $1/k_c$ in Eqn. (28) below.

Line 1 corresponds to Eqn. (20), line 2 to Eqn. (22), and lines 3 and 4 to the constraints $R \gtrsim R_p$ and $R \gtrsim R_f$. All these are for the very weak field $B = 10^{-4}$ T; the dashed lines show the corresponding constraints for $B = 10^{-2}$ T.

Finally, we may use the solution in Eqn. (13) to find the temperatures at $s = 1$ which are consistent with $T(\rho) = T_\rho = 10^5$ K. These are displayed in Fig. 2. As can be seen there, constraining $T(1)$ to lie between 10^4 and 5×10^4 K puts a tight constraint on the possible values of (ρ, RB) . That is, only a narrow range of these parameters allow solutions with a cool outer shell in this T range.

3.2 Anomalous enhancements to κ_\perp

The length scales in this model are consistently very small, due to the extreme smallness of the perpendicular conduction coefficient κ_\perp in Eqn. (4). Observations in tokamaks consistently show thermal (and mass) transport coefficients enhanced by one or two orders of magnitude over the classical values. This is generally attributed to micro-turbulence (small-scale, low-frequency, and more likely in the magnetic than the electric field), or to some other small-scale quasi-diffusive process, though theoretical ex-

Figure 2a and b are in files figtaua.ps and figtaub.ps

Figure 2: The values of $T(1)$ which are consistent with a boundary temperature $T(\rho) = 10^5$ K, for various values of the parameters ρ and RB , taken from Eqn. (13) ($\beta = 2$). The contours shown on the surface are at 10^4 K and 5×10^4 K. Fig. 2b shows the same two contours for $\beta = 2$ as well as the numerical solutions of the cases $\beta = 1.9$ (dotted line) and the polynomial fit (dashed line). Finally, the dot-dash line shows the contours for the case $B = \text{constant}$, discussed in Sect. 5 below.

planations are inconclusive (Liewer, 1985). Though we should, of course, be wary of exporting these observations from the laboratory to the more tenuous and cooler atmosphere of the sun, it seems safe to assume that the scales derived here are substantial underestimates. Effective enhancement of κ_1 will proportionately enhance the right-hand sides of Eqns. (20), (22) and (14) and raise the corresponding upper boundaries in Fig. 1. In the Sun, ion-sound turbulence can result in a collision time much smaller than that given above (Priest, 1984), but this occurs only when the field has scales below $\sim 10^{21}(B/\text{T})(n_e/\text{m}^{-3})^{-1}(T_e/\text{K})^{-1/2}$ m, comparable with the lower limits given in Fig. 1.

If we take a coronal magnetic field of 10^{-2} T and a boundary temperature of $T_\rho = 10^5$ K, then Fig. 1 and the above two-order enhancement in the length scales give a filament scale of 10^2 m, and hence require of the order of 10^9 filaments to fill up a flare area of 10^{13} m².

3.3 Numerical solution of the temperature profile

For comparison with the above analytic results, and to confirm that our analytic solution is not somehow pathological, we have also solved Eqn. (7) numerically, with $f(T)$ represented by power-laws with various values of β ,

and by a polynomial fit (Keddie, 1970) to the Cox-Tucker curve. The solutions are dependent on β , but not unduly so, so that we can be confident that our results are qualitatively insensitive to the precise form of the loss-function. See Fig. 2.

3.4 Comparison with the longitudinal temperature profile

In section 2 above, we stated that we can ignore the effects of longitudinal conduction, and we can now check that this is indeed true. We can avoid solving the full 2-d problem in the (s, z) plane, and instead calculate what the longitudinal temperature gradient would have to be for this assumption to fail. Writing the longitudinal flux density $Q_\parallel(z)$ as

$$Q_\parallel(z) = \kappa_\parallel^0 T^{5/2} T_z, \quad (23)$$

where the coefficient $\kappa_\parallel^0 = (2.23 \times 10^{-9} \text{ W m}^{-1} \text{ K}^{-7/2}) / \ln \Lambda$ (Spitzer, 1962), and $T_z \equiv dT/dz$, we can use the solution in Eqn. (12) to write the ratio of transverse and longitudinal flux densities as

$$\frac{Q_\perp}{Q_\parallel} = \frac{\chi a^2 R}{2\kappa_\parallel^0} \left(\frac{1}{s} - s \right) \left(\frac{3}{2} \tau(s) \right)^{5/3} \frac{1}{T_z}, \quad (24)$$

where $\tau(s)$ is given by Eqn. (13).

In Fig. 3, we show $\tilde{T}_z(s)$, defined so that $Q_\perp > Q_\parallel$ for $T_z < \tilde{T}_z$, for a variety of radii and magnetic field strengths. Note that \tilde{T}_z decreases towards the surface of the core $s = \rho$: the increase in temperature here enhances the longitudinal conduction coefficient as it suppresses the transverse one. Note also that \tilde{T}_z increases with increasing R : this is initially surprising, as one would expect the assumption $Q_\perp \gg Q_\parallel$ to be more comfortably satisfied with finer filamentation; instead this reflects the steepening of the transverse gradient with larger R , which was noted above Eqn. (17). In all cases, $\tilde{T}_z(s=1) = 0$, reflecting the boundary condition on Eqn. (11) of $dT/ds|_{s=1} = 0$, and we should also take $T_z(s=\rho) = 0$ since, by hypothesis, the temperature at the surface of the core is a constant (Eqn. (1)). It is difficult to estimate T_z in a model-independent way, but if we take $T_z = (10^9 \text{ K})/l \sim 1 \text{ K m}^{-1}$, we can see that for this geometry, the transverse flux density should dominate over the longitudinal one everywhere except at the edge of the shell, $s = 1$.

Figure 1 is in file w7173.ps

Figure 3: Values of $\tilde{T}_z(s)/\text{K m}^{-1}$, the values of the longitudinal temperature gradient $T_z(s)$ which make the transverse and longitudinal heat flux densities equal, as obtained from Eqn. (24). The curves are labelled by $(\log_{10}(R/\text{m}), \log_{10}(B/\text{T}))$.

4 Stability

We have considered only a static solution to the energy-balance problem. To determine the stability of these solutions, we must examine the heat equation

$$\frac{1}{\gamma-1} \frac{dp}{dt} - \frac{\gamma}{\gamma-1} \frac{p}{n} \frac{dn}{dt} + K(n, T) + L(n, T) = 0. \quad (25)$$

The flux term $K(n, T)$ is

$$\begin{aligned} K(n, T) &= -\nabla \cdot (\kappa_{\perp} \nabla T) \\ &= -\frac{1}{r} \kappa_0 \frac{d}{dr} \left[r n^2 T^{-1/2} \frac{dT}{dr} \right], \end{aligned}$$

and the radiative loss function $L(n, T) = n^2 f(T)$, so that $rK(n, T) + rL(n, T) = 0$ is our Eqn. (7).

We combine this with the equation of state, the equation of motion, and the mass continuity condition, and perturb the equilibrium T , ρ and p by terms of the form

$$a(r, t) = a_0 + a_1 \exp(\nu t + ikr).$$

For simplicity, we will *separately* consider the limiting cases of isobaric ($p_1 = 0$) and isochoric ($\rho_1 = 0$) perturbations – the former match the conditions in the earlier

part of our analysis, but the latter are more appropriate for perturbations with timescales shorter than the sound travel time.

4.1 Isobaric perturbations

For isobaric perturbations, $p_1 = 0$, the linearised heat equation, Eqn. (25), becomes

$$-\frac{\gamma}{\gamma-1} \nu \frac{p_0}{n_0} n_1 - a^2 \kappa_0 T_0^{-5/2} i k T_1 [1/r + i k] + L_T T_1 + L_n n_1 = 0, \quad (26)$$

where L_T and L_n denote partial derivatives of the heating function $L(n, T)$. Combining this with the equation of state $a = nT$, the linearised perturbed equation of state

$$\frac{n_1}{n_0} - \frac{T_1}{T_0} = 0,$$

the pressure $p_0 = k_B a$ (where k_B is Boltzmann's constant), setting $f(T) = \chi T^{\beta}$, and setting $\gamma = 5/3$, gives the real part of the growth parameter ν as

$$\text{Re } \nu = -\frac{2}{5} \frac{a}{k_B} \left(\kappa_0 T_0^{-3/2} k^2 - (2 - \beta) \chi T_0^{\beta-2} \right), \quad (27)$$

which is negative when $\beta < 2$ and $k > k_c$, for a critical wavenumber k_c defined by

$$k_c^2 = (2 - \beta) \frac{\chi}{\kappa_1} B^2 T_0^{\beta-1/2}. \quad (28)$$

Thus, solutions to Eqn. (7) are stable to isobaric perturbations with a scale *shorter* than $1/k_c$. This applies also to more general forms of the loss function $f(T)$, which can be locally approximated by a power-law T^{β} . The dependence of $1/k_c$ on T and B is shown in Fig. 1.

4.2 Isochoric perturbations

The isochoric case $\rho_1 = 0$ is more appropriate for perturbations which are rapid compared to the sound time. The same analysis as above produces a growth rate

$$\frac{1}{\gamma-1} \frac{p_0}{T_0} \text{Re } \nu = -n_0^2 (\beta \chi T_0^{\beta-1} + \kappa_0 T_0^{-1/2} k^2), \quad (29)$$

which is certainly negative as long as β is positive.

We can conclude that the only unstable perturbations are long-wavelength isobaric ones, which do not generate sufficiently steep temperature gradients, though these perturbations should be stabilised when the perturbation

has heated enough to provide a sufficiently large gradient. Note that the solution outlined above Eqn. (17), where the temperature gradient in the inner part of the shell would have a constant maximum value, would be unstable also – since any local temperature enhancement will not result in a stabilising increase in the gradient, the temperature increase will go unchecked.

5 Results in low-beta plasmas

The full pressure-balance condition is

$$n(s)T(s) + \frac{B^2}{2k_B\mu_0} = a', \quad (\text{constant}) \quad (30)$$

which we reduced to Eqn. (5) by taking the magnetic field to be constant across the filament. The two terms in Eqn. (30) are equal (for $a = 10^{23} \text{ K m}^{-3}$) when $B \approx 1.9 \times 10^{-3} \text{ T}$. If B is small compared with this, the condition in Eqn. (30) reduces to Eqn. (5). For a low- β plasma, on the other hand, a change in the gas pressure can be compensated by a tiny change in the magnetic field, and Eqn. (30) reduces to $B = \text{constant}$ (we thank the referee, Dr. Schüßler, for emphasising this point).

In this latter case, given that the filament length and total flux are constant, and the field lines are frozen to the plasma, we can further take $B/n = \text{constant}$, and write Eqn. (7) as

$$\frac{d}{ds}(s\tilde{\tau}') - \frac{B^2 R^2 \chi}{\kappa_1} s \left(\frac{\tilde{\tau}}{2}\right)^{2\beta} = 0, \quad (31)$$

where $\tilde{\tau} = 2\sqrt{T}$ (compare Eqn. (11)); the numerical solution to this is displayed in Fig. 2. The solutions in this case are remarkably similar to the solutions resulting from the approximation in Eqn. (5). This is explained by noting that the change from Eqn. (5) to Eqn. (30) affects *both* the radiative and conductive losses: with constraint Eqn. (5), the radiative loss term in Eqn. (7) is $\sim T^{\beta-2}$, and the conduction coefficient in Eqn. (4) is $\sim T^{-5/2}$; with Eqn. (30), these become T^β and $T^{-1/2}$ respectively. This means that as the plasma temperature rises above the equilibrium, both radiative and conductive cooling are more efficient for the latter case than for the former.

Because we have explicitly considered an isochoric model here, only the stability analysis in Sect. 4.2 is relevant, and we can see that the solution for this case is stable (Eqn. (29)) for all positive β .

6 Conclusion

We have presented a simple model of heat transport *across* the magnetic field in a highly filamented solar flare. This is analytically soluble in a simple case, which numerical solution shows to be representative.

If we assume that the various potential complications do not qualitatively change the results, then the analysis very directly leads to predicted maximum cell sizes (Fig. 1) which, though smaller than any yet observed, are larger than the lower limits imposed by the plasma conditions. Current uncertainties in our understanding of anomalous enhancements to the cross-field conduction strongly suggest that our upper limits on the cell size are underestimates.

The upper limits we have derived here are strongly dependent on temperature and field strength, so that in the cooler and less magnetic environment of prominences, we would expect this mechanism to have a more prominent role than in flares.

Given that the mechanism described here is important in flares of all sizes, then the dependence of ρ on P_A in Eqn. (16), and the interdependence of ρ , and RB in Fig. 2 are testable predictions. Even if we do not believe that this mechanism governs flare geometry as generally as this, we must regard the scale limits as delimiting a domain within which the effects of cross-field thermal conduction dominate over longitudinal conduction, and cannot be ignored.

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